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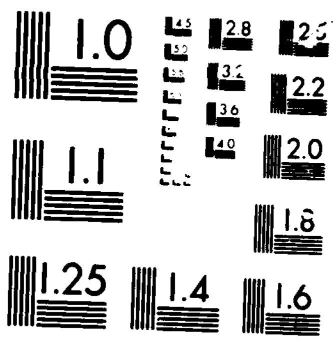

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High Gain Free Electron Laser Oscillators

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We have carried out a fully relativistic, nonlinear analysis of the spatial and temporal evolution of multiple modes within a free electron laser oscillator. The analysis is both analytical and numerical, and is conducted self-consistently within the framework of the Vlasov-Maxwell system of equations, including the collective effects produced by the longitudinal space-charge wave potential. The analytic calculation of the small signal gain yields a dispersion relation for the moderate to high gain ($\Gamma L > 1$) operation of the free electron laser. The numerical simulations verify the analytic results for the linear gain regime and predicts high efficiencies for the nonlinear saturated state of the radiation fields.

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CONTENTS

HIGH GAIN FREE ELECTRON LASER OSCILLATORS

I. Introduction

We have conducted an analytical and numerical analysis of the field evolution in a high gain free electron laser operating in the oscillator configuration, as depicted in Fig. 1. The analysis is applicable to systems with electron beam pulse lengths which are longer that the particle transit time in the resonator. The electron beam equilibrium is therefore assumed to be spatially uniform and temporally stationary. The radiation field and phase averages which are performed with the ensemble of electrons is conducted for an interaction length which consists of the entire wiggler structure. This is in contrast to other simulations (theories) which perform the ensemble average over the wavelength of the ponderomotive potential; as is applicable to systems with temporally stationary fields[1] or short beam pulses[2] that are spatially periodic.

We find that the numerical simulations yield qualitative and quantitative agreement with the theory. The theory for the example given (strong pump Compton regime) can be separated into three operation regimes which we shall denote as the ultra-high gain, moderate gain and low gain regimes. Both the ultra-high gain ($\Gamma_k L >> 1$) and the low gain ($\Gamma_k L << 1$) regimes yield growth rates that exhibit the same scaling with beam current, energy and wiggler field as is obtained for an FEL amplifier operating in these regimes. Additionally, we consider a moderate gain regime ($\Gamma_k L \geq 1$) which is of direct interest to NRL experimental parameters.

II. Theoretical Model

An analysis of the space time evolution of the fields and particles within an FEL oscillator requires a self-consistent coupling of the fundamental equations for the particles

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and fields[3-8]. We have considered a Maxwell-Vlasov description of the fields and particles. The analysis in Appendix A results in the following system of equations for the coupling of the fields and particles. The backward travelling wave evolves according to, $\partial \tilde{a}_b(z)/\partial z - i\alpha\tilde{a}_b(z) = 0$. The forward travelling potential and the electrostatic potential evolve according to,

$$\begin{aligned}
&\frac{\partial}{\partial z}\tilde{a}_f(z) + i\alpha\tilde{a}_f(z) = \\
&\quad - c_1\int_0^z dz'(z'-z)\exp[-i\Delta K(z'-z)](\partial/\partial z' - iK)[\beta_w\tilde{a}_f(z')/2 - \tilde{\phi}(z')] \\
&\quad + ic_2\int_0^z dz'\exp[-i\Delta K(z'-z)](\partial/\partial z' - iK)[(1+\beta_{z0}^2)\beta_w\tilde{a}_f(z')/2 - \tilde{\phi}(z')], \qquad (1)
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial}{\partial z}\tilde{\phi}(z) - iK\tilde{\phi}(z)/2 = \\
&\quad - c_3\int_0^z dz'(z'-z)\exp[-i\Delta K(z'-z)](\partial/\partial z' - iK)[\beta_w\tilde{a}_f(z')/2 - \tilde{\phi}(z')] \\
&\quad + ic_4\int_0^z dz'\exp[-i\Delta K(z'-z)](\partial/\partial z' - iK)[\beta_w\tilde{a}_f(z')/2 - (1-\beta_{z0}^2)\tilde{\phi}(z')]. \qquad (2)
\end{aligned}$$

The parameters in Eqs. (1) and (2) are given by, $\alpha = \Delta\omega/c - \omega_p^2(1-\beta_w^2)/2\omega_0 c\bar{\gamma}$, $c_1 = c_2(1-\beta_{z0}^2)(\omega_0+\Delta\omega)/v_{z0}$, $c_2 = \omega_p^2\beta_w/2\omega_0 c\beta_{z0}^2\bar{\gamma}$, $c_3 = c_1\omega_0/\beta_w K c$, $c_4 = c_3/(1-\beta_{z0}^2)$, $K = k_0 + k_w$ and $\Delta K = K - (\omega_0+\Delta\omega)/v_{z0}$. By making use of the convolution theorem, the Laplace transform of Eqs. (1) and (2) yields,

$$\begin{aligned}
&\{(s+i\alpha)[(s-i\Delta K)^2 + 2c_3] + iKc_1\beta_w/2\}\hat{a}_f(s) = \\
&\qquad\qquad \{(s-i\Delta K)^2 + 2c_3(1-\beta_w^2/4)\}\tilde{a}_f(0), \qquad (3)
\end{aligned}$$

$$\{(s-i\Delta K)^2 + 2c_3\}\hat{\phi}(s) = c_3\beta_w[\hat{a}_f(s) - i\tilde{a}_f(0)/K]. \qquad (4)$$

where $\hat{a}_f(s)$ and $\hat{\phi}(s)$ are the Laplace transformed vector and scalar potentials, and we have retained only the terms in the driving current that arise from the momentum derivatives of the phase, $\partial\Delta K/\partial p_z$. We have also assumed that the electron beam enters the resonator unbunched so that $\tilde{\phi}(z=0) = 0$.

Since the singularities of the Laplace transformed potentials are isolated poles, the Bromwich inversion of these transforms can be easily performed.

$$\tilde{\phi}(z) = \sum_j \text{Residue}\{\hat{\phi}(s), s_j\} \exp(s_j z), \tag{5}$$

$$\tilde{a}_f(z) = \sum_j \text{Residue}\{\hat{a}_f(s), s_j\} \exp(s_j z), \tag{6}$$

where s_j, are the poles of the Laplace transformed potentials. The solution for the radiation potential given in Eq. (6) is of the same generic form as the solution obtained by Bernstein and Hirshfield[9] for the FEL amplifier configuration. Our analysis shall differ in that the backward travelling waves are not neglected, and the combination of the forward and backward travelling waves are required to satisfy the appropriate boundary conditions at the mirror surfaces. Specifically, the tangential component of the electric field must be zero at the non-transmitting mirror surface, i.e., $\tilde{a}_f(0) + \tilde{a}_b(0) = 0$. At the partially transmitting mirror surface at the end of the resonator, the tangential components of the electric field must be continuous, i.e., $\tilde{a}_f(L)\exp(-ik_0L) + \tilde{a}_b(L)\exp(ik_0L) - (1 - \sqrt{R})\tilde{a}_f(L)\exp(-ik_0L) = 0$, where L is the length of the resonator and R is the fractional power reflected from the far end resonator mirror. This yields the following expression for the boundary conditions at the mirror surfaces,

$$\exp\frac{i\Delta\omega L}{c} = \frac{\sqrt{R}}{\tilde{a}_f(0)} \exp\{i[\frac{\omega_p^2}{2\omega_0 c\bar{\gamma}}(1 - \beta_w^2/2) - 2k_0]L\} \sum_j \text{Residue}\{\hat{a}_f(s), s_j\} \exp(s_j L), \tag{7}$$

which is the equation that self-consistently determines the complex operating frequency, $\Delta\omega$, of the oscillator.

III. Results for Compton Regime

In the Compton regime the effect of the electrostatic potential can be neglected[10,11]. In addition we shall assume that the spatial derivative of the vector potential in the driving current is negligible. These derivatives are negligible when, $|(s - i\Delta K)^2| >> 2c_3$ and $|s| << K$. Under these conditions the Laplace transform of the vector potential is given by,

$$\hat{a}_f(s) = \tilde{a}_f(0)(s - i\Delta K)^2 / \prod_{j=1}^{3}(s - s_j), \tag{8}$$

where s_j are the roots of the dispersion relation, $(s - i\Delta K)^2(s + i\alpha) = -ic_1\beta_w K/2 \approx -i\omega_p^2(1 + \beta_{z0})\beta_w^2 k_w / 4\beta_{z0}^3 c^2\overline{\gamma}$. Since k_0 is a free parameter, choose k_0 such that, $\Delta K = k_0 + k_w - (\omega_0 + \Delta\omega)/v_{z0} = -\Delta\omega/c + \omega_p^2(1 - \beta_w^2/2)/2\omega_0 c\overline{\gamma}$. Which results in the following solutions to the dispersion relation,

$$s_j = -i\left[\frac{\Delta\omega}{c} - \frac{\omega_p^2}{2\omega_0 c\overline{\gamma}}(1 - \beta_w^2/2)\right] + \Gamma_0 \begin{cases} i2/\sqrt{3} \\ 1 - i/\sqrt{3} \\ -1 - i/\sqrt{3} \end{cases} . \tag{9}$$

where $\Gamma_0 = \sqrt{3}k_w[\omega_p^2(1 + \beta_{z0})\beta_w^2/4k_w^2c^2\overline{\gamma}]^{1/3}/2\beta_{z0}$ is a spatial growth rate corresponding to the largest spatial growth rate in the amplifier case[12]. By evaluating the residues of $\hat{a}_f(s)$ for each of these poles, one obtains the following solution for the spatial structure of the radiation potential,

$$\tilde{a}_f(z) = \frac{\tilde{a}_f(0)}{3}\sum_{j=1}^{3}\exp(s_j z). \tag{10}$$

It is evident from Eq.(10) that the spatial growth of the radiation field can be described by the interference of three modes; which can be identified as the positive and negative energy beam modes, and a transverse electromagnetic mode. The constructive or destructive nature of this interference is dependent on the values of the physical parameters which characterize the roots, s_j. For physical parameters such that, $\Gamma_0 L >> 1$, the unstable mode dominates and one obtains exponential spatial growth at the rate Γ_0.

The temporal growth rate of the radiation field is obtained from the negative imaginary part of the complex oscillator frequency, $\Delta\omega$. The oscillator frequency is determined by the boundary conditions as expressed in Eq. (7), which for the approximate roots under consideration yields,

$$\exp\frac{i\Delta\omega L}{c} = \frac{\sqrt{R}}{3}\exp\left\{i\left[\frac{\omega_p^2}{2\omega_0 c\overline{\gamma}}(1 - \beta_w^2/2) - 2k_0\right]L\right\}\sum_{j=1}^{3}\exp(s_j L). \tag{11}$$

We shall consider three distinct solutions to this equation for the complex oscillator frequency. The first of which is the ultra-high gain regime ($\Gamma_0 L >> 1$), in which case, only the fastest growing mode in Eq. (11) is retained. The second case is the moderate gain regime ($\Gamma_0 L \geq 1$), where only the decaying mode in Eq. (10) is neglected. The final case is valid for arbitrary gain and all terms in Eq. (11) are retained. The imaginary part of the oscillator frequency yields the following temporal growth rates,

$$\Gamma_\omega \frac{L}{c} = \frac{1}{2} \ln \frac{\sqrt{R}}{3} + \Gamma_0 L/2, \qquad \text{Ultra-High Gain} \qquad (12)$$

$$\Gamma_\omega \frac{L}{c} = \frac{1}{2} \ln \frac{\sqrt{R}}{3} + \frac{1}{4} \ln[2\cosh(\Gamma_0 L) + 2\cos(\sqrt{3}\Gamma_0 L)], \qquad \text{Moderate Gain} \qquad (13)$$

$$\Gamma_\omega \frac{L}{c} = \frac{1}{2} \ln \frac{\sqrt{R}}{3} + \frac{1}{4} \ln[1 + 4\cos(\sqrt{3}\Gamma_0 L)\cosh(\Gamma_0 L) + 4\cosh^2(\Gamma_0 L)]. \qquad \text{Arbitrary Gain} \qquad (14)$$

In each of the expressions for the growth rate the first term is negative definite. This represents the effect of losses at the mirrors and the coupling losses due to the splitting of the radiation into three modes. The necessary condition for the oscillator to lase is that the remaining terms exceed this loss. For the ultra-high gain case this requires $\Gamma_0 L \geq -\ln(\sqrt{R}/3)$. This expression has been confirmed experimentally[13] in recent operation of the NRL FEL oscillator. The interaction length, L, can be varied by dumping the beam at different axial locations within the wiggler. For the following set of experimental parameters, beam energy $E_0 = 500\ keV$, beam current $I = 100\ A$, wiggler field strength $B_w = 615\ G$, beam radius $r_b = 0.64\ cm$ and wiggler length $\ell_w = 4.0\ cm$, the minimum interaction length is determined to be 45 cm. Inserting this value into Eq.(12) yields a theoretical value of 0.64 for the reflection coefficient. The independently measured Bragg reflection coefficient has the value 0.65, which is in excellent agreement with the theoretical value.

IV. Multi–Mode Simulation

The space-time evolution of the fields in the resonator is simulated by numerically evolving the equations for the fields and particles. The radiation field model for the multi-mode simulation is given by,

$$\vec{A}_R(z,t) = \sum_n a_n(t) \sin(k_n z) \exp(i\omega_n t) \hat{e}_- + c.c., \tag{15}$$

$$\phi(z,t) = \sum_n \phi_{1n}(t) \sin[(k_n + k_w)z - \omega_n t] + \phi_{2n}(t) \cos[(k_n + k_w)z - \omega_n t], \tag{16}$$

where $k_n = n\pi/L = \omega_n/c$ and the sum is over the discrete number of modes under consideration. This model has the property that the complex expansion coefficients in the harmonic analysis, $a_n(t), \phi_{1n}(t), \phi_{2n}(t)$, are only functions of time; which results in ordinary differential equations for the particle and field evolution. This model also has the attribute that the field boundary conditions at two perfectly reflecting mirrors is automatically satisfied, and we model the resonator losses heuristically by adding a damping term to the wave equation, $[\partial^2/\partial z^2 - c^{-2}\partial^2/\partial t^2 - \nu c^{-2}\partial/\partial t]\vec{A}_R(z,t) = 4\pi c^{-1}\vec{J}_\perp(z,t)$, where $\nu = \omega/Q$ and Q is the quality factor of the resonator. The driving currents and charge densities for the vector and scalar potentials are modeled with a discrete distribution function as follows,

$$\rho(z,t) = -e\int dz_0 n_0(t)\delta(z - \bar{z}(z_0,t)), \tag{17}$$

$$\vec{J}_\perp(z,t) = -\frac{e^2\vec{A}}{mc}\int dz_0 n_0(t)\delta(z - \bar{z}(z_0,t))/\gamma_0, \tag{18}$$

where, $\vec{A} = \vec{A}_R + \vec{A}_w$, $n_0(t) = n_0(\infty)[1 - \exp(-t/t_R)]$ and $n_0(\infty)$ is the flattop density of the electron beam pulse. t_R is the characteristic rise time for the beam current or density, and $\bar{z}(z_0,t)$ is the axial orbit of a particle located at position z_0 at $t = 0$.

The slowly varying field approximation, $\partial a_n(t)/\partial t << \omega_n a_n(t)$, yields the following set of equations for the evolution of the fields and particles.

$$\hat{A}_n' = -\frac{\nu^* \hat{A}_n}{2} - \beta_{1n} \int_{\tau-1}^{\tau} d\tau_0 \sin[\tilde{\psi}_n(\tau_0,\tau)] \frac{\overline{\gamma}}{\gamma_0} P(\tau_0), \tag{19}$$

$$\theta_n' = -\frac{\nu_I^*}{2} + \frac{\beta_{1n}}{\hat{A}_n} \int_{\tau-1}^{\tau} d\tau_0 \cos[\tilde{\psi}_n(\tau_0,\tau)] \frac{\overline{\gamma}}{\gamma_0} P(\tau_0), \tag{20}$$

$$\hat{\phi}_{1n} = -\beta_{2n} \int_{\tau-1}^{\tau} d\tau_0 \cos[\tilde{\psi}_n(\tau_0,\tau)] P(\tau_0), \tag{21}$$

$$\hat{\phi}_{2n} = -\beta_{2n} \int_{\tau-1}^{\tau} d\tau_0 \sin[\tilde{\psi}_n(\tau_0,\tau)] P(\tau_0), \tag{22}$$

$$\begin{aligned} \tilde{\psi}_n'' = & -\theta_n'' + \beta_{3n} \sum_m (k_m + k_w) L [\hat{\phi}_{2m} \cos[\tilde{\psi}_m(\tau_0,\tau)] - \hat{\phi}_{1m} \sin[\tilde{\psi}_m(\tau_0,\tau)]] \\ & + \beta_{4n} \sum_m \Big\{ [(k_m + k_w)L - k_m L \beta_z + \beta_{0z}\beta_z\theta_m'] \hat{A}_m \sin[\tilde{\psi}_m(\tau_0,\tau)] \\ & - \beta_{0z}\beta_z \hat{A}_m' \cos[\tilde{\psi}_m(\tau_0,\tau)] \Big\}. \end{aligned} \tag{23}$$

We have introduced the following normalized parameters, $\tau = v_{0z}t/L$, $\hat{A}_n = eA_n/mc^2$, $\hat{\phi} = e\phi/mc^2$, $(...)' = \partial(...)/\partial\tau$. We have also made the following definitions, $a_n = -i\hat{A}_n(\tau)\exp(i\theta_n\tau)$, $\tilde{\psi}_n(\tau_0,\tau) = (k_n + k_w)z(\tau_0,\tau) - \omega_n L\tau/v_{0z} - \theta_n(\tau)$, $\nu_R^* = \nu L/v_{0z}$, $\nu_I^* = \omega_p^2 k_w LF(1 - \beta_w^2/2)/2k_w^2 c^2 \overline{\gamma}$, $\beta_{1n} = F\beta_w L\beta_0\overline{\gamma}\omega_p^2/2c^2\beta_{0z}^2 k_n$, $\beta_{2n} = 2F\omega_p^2\beta_0/c^2(k_n + k_w)^2\beta_{0z}$, $\beta_{3n} = L(k_n + k_w)/\overline{\gamma}\gamma_z^2\beta_{0z}^2$, $\beta_{4n} = \beta_w(k_n + k_w)L\overline{\gamma}/2\beta_{0z}^2\gamma_0^2$ and $P(\tau_0) = 1 - \exp(-\tau_0/\tau_R)$, where L is the length of the resonator, F is the filling factor and ω_p is the nonrelativistic plasma frequency.

This system of equations is solved numerically by using a four-point Adams–Bashforth predictor corrector scheme which is initialized by using the three point Runge–Kutta method. The ensemble average over initial electrons, $\int_{\tau-1}^{\tau}(...)d\tau_0$, is typically performed with two thousand (2000) particles. The results of the simulations and the linear theory, obtained from the linearization of Eqs. (19) - (23), are shown in Figs. 2 and 3.

V. Conclusions

A comparison of the temporal growth rates obtained from the linear theory and the numerical simulations is shown in Figs. 2 and 3. The growth rates for the simulations are obtained numerically from the field amplitude data during the initial field evolution, where

the wave growth is linear. In Fig. 2 the data is presented for a low gain case with physical parameters given by, $\gamma = 2.0, I = 5A, F = 0.2. k_w r_b = 0.62831, \vartheta_w = 0.2$ and $L/\ell_w = 50$. There is an excellent agreement between theory and simulation. In Fig. 2 we also compare the theoretical and numerical efficiencies for a high gain case. Where the efficiency is defined as the stored electromagnetic energy density normalized to the incident beam energy density. The theoretical estimates of the efficiency are based on particle trapping arguments[12], with the assumption that all the energy lost by the particles is converted into electromagnetic energy. The characteristic change in velocity of a particle is given by the difference in the beam velocity and the phase velocity of the trapping potential. This phase velocity is approximated from the results of the linear dispersion relation. Again we find good qualitative and quantitative agreement between the simulation and theory.

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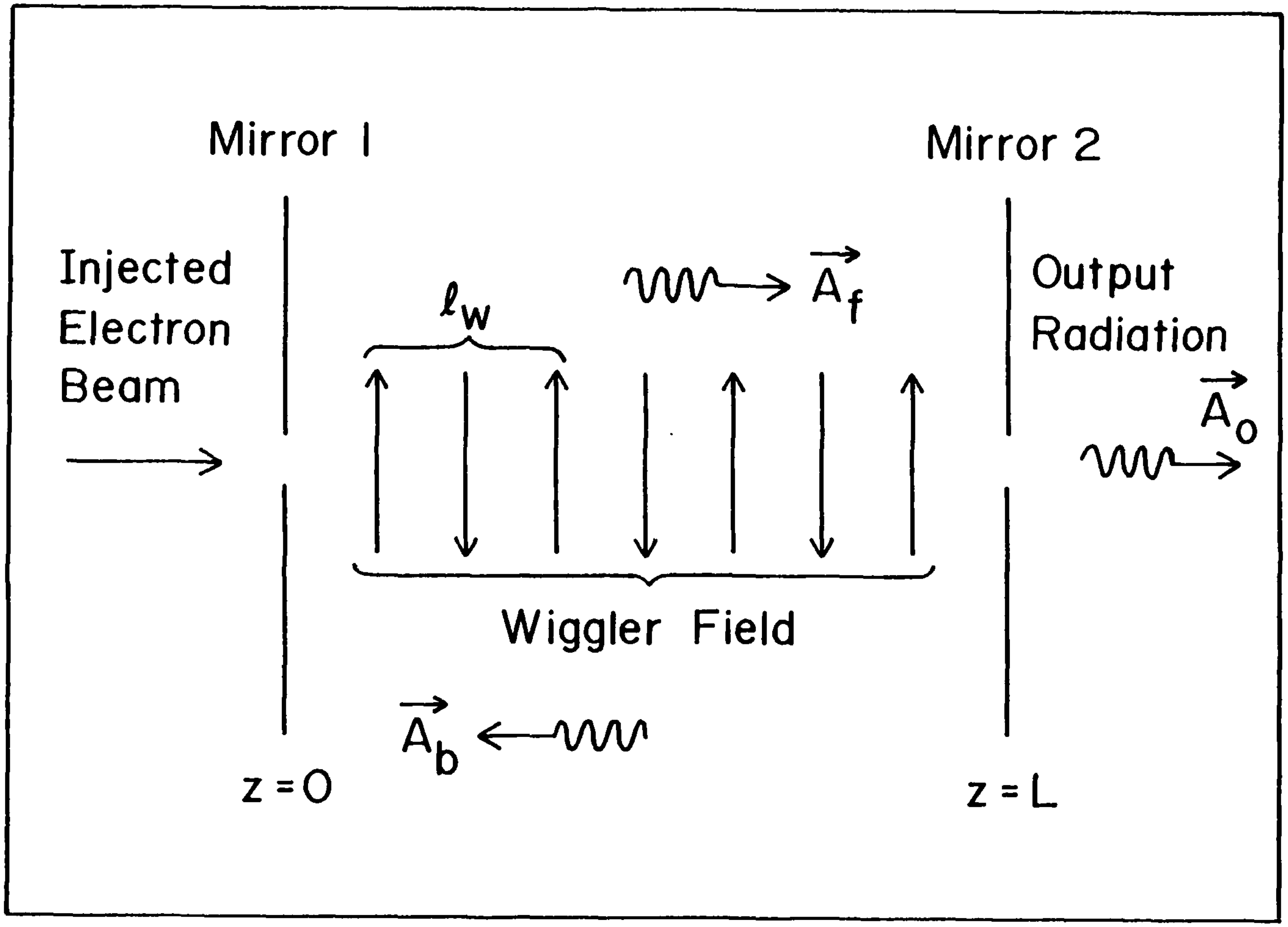

Figure 1. Schematic of high gain FEL oscillator, where $\vec{A}_f = a_f(z,t)\exp(-ik_0 z + i\omega_0 t)\hat{e}_- +$ c.c. is the forward propagating wave, $\vec{A}_b = a_b(z,t)\exp(ik_0 z + i\omega_0 t)\hat{e}_- +$ c.c is the backward propagating wave and $\vec{A}_0 = (1-\sqrt{R})\vec{A}_f$ is the transmitted wave.

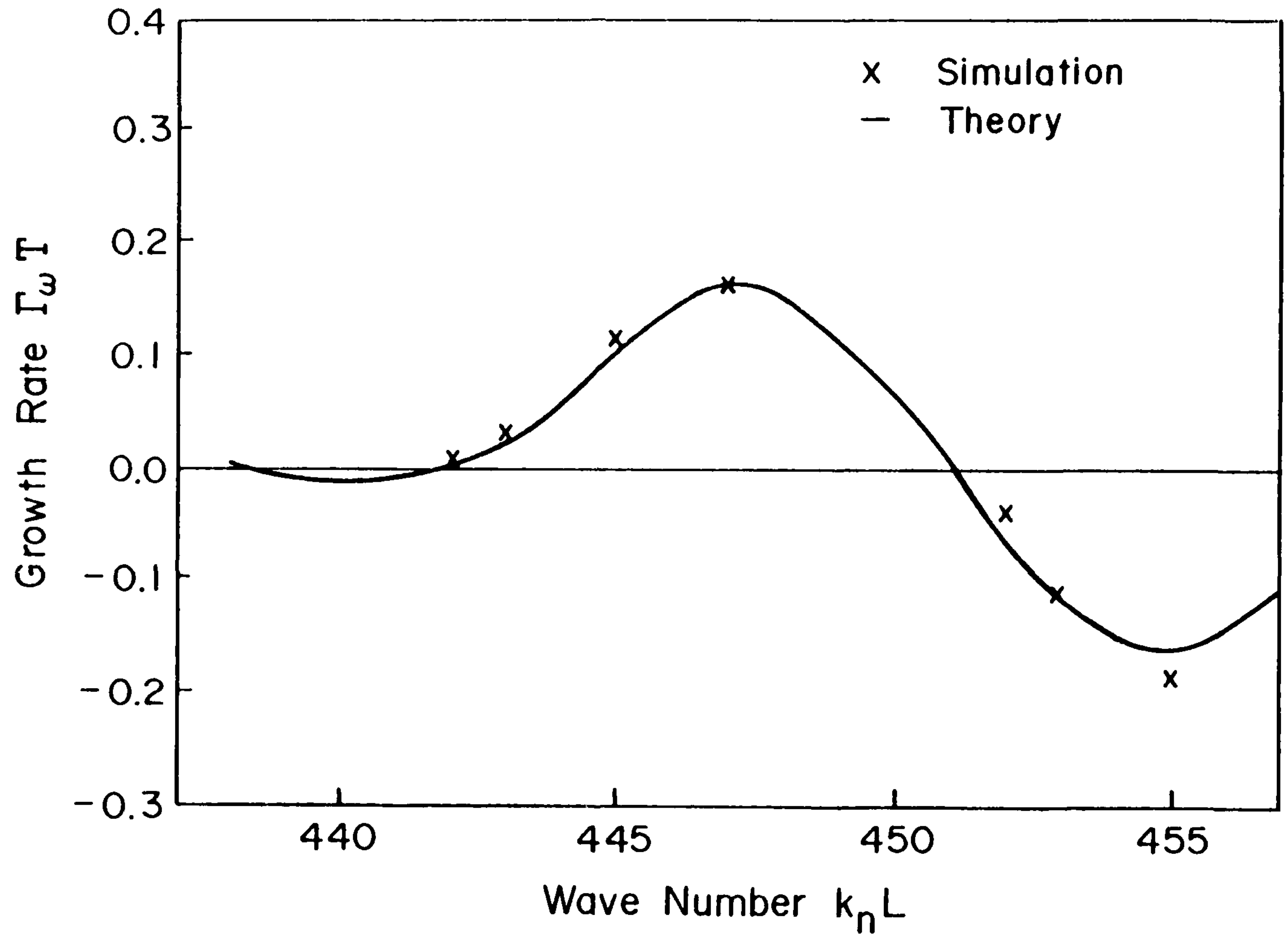

Figure 2. Comparison of the temporal growth rate from theory and numerical simulation in low gain Compton regime. The following physical parameters are used: $\gamma = 2.0$, $I = 5A$, $F = 0.2$, $k_w r_b = 0.62831$, $\beta_w = 0.2$ and $L/\ell_w = 50$.

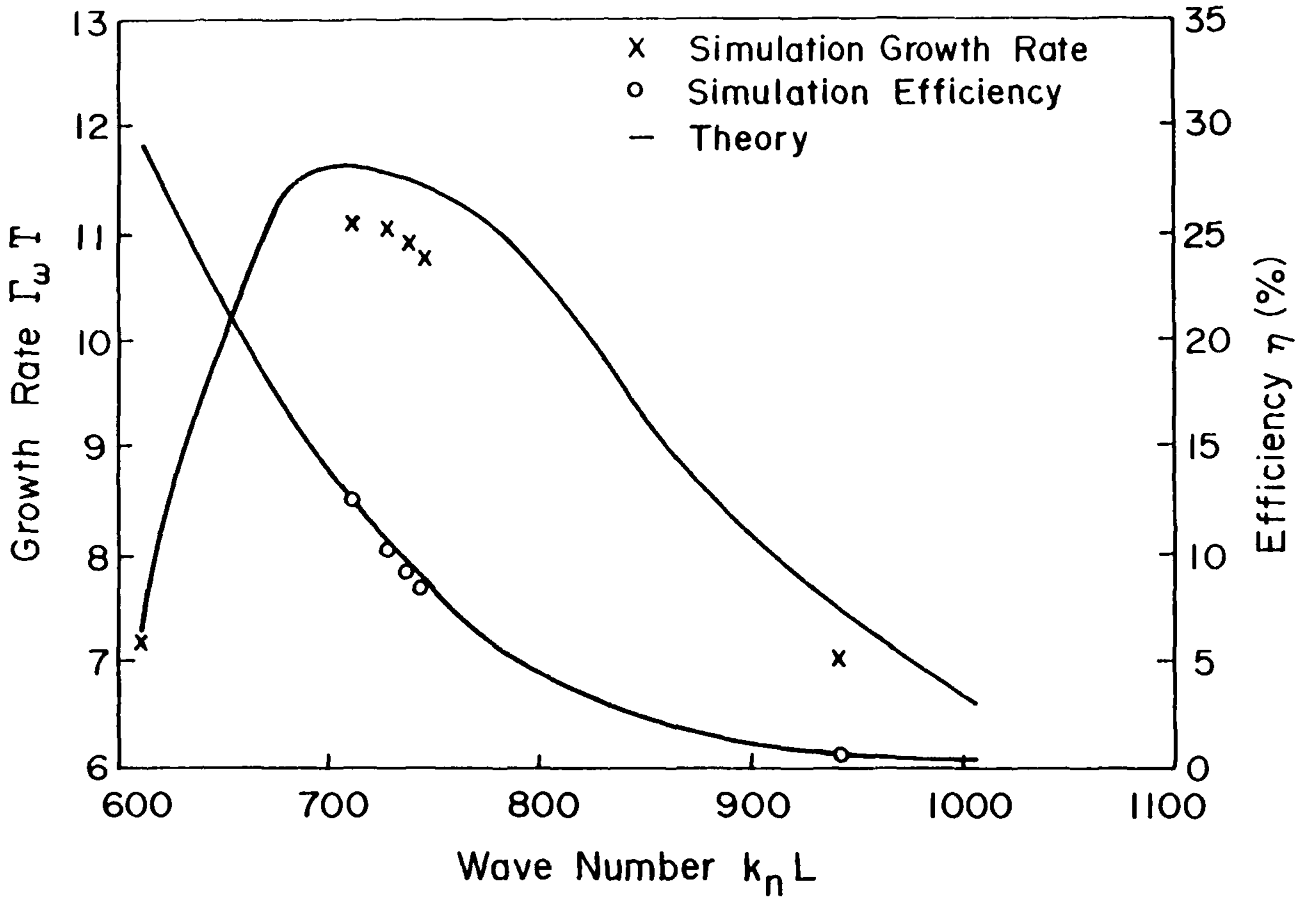

Figure 3. Comparison of the temporal growth rate and saturation efficiency from theory and numerical simulations in high gain Compton regime. The following physical parameters are used: $\gamma = 2.5$, $I = 500A$, $F = 0.2$, $k_w r_b = 0.7853$, $\beta_w = 0.3686$ and $L/\ell_w = 22$.

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Appendix A

In the following, we shall consider the space-time evolution of the radiation fields produced by the interaction of a beam of relativistic electrons with a helical wiggler field contained within the mirrors of an optical resonator. The analysis is fully relativistic and is conducted self-consistently within the framework of the Vlasov-Maxwell system of equations.

The wiggler vector potential is modeled as follows,

$$\vec{A}_w(z) = \frac{B_w}{k_w}\left[\exp(ik_w z)\hat{e}_- + \exp(-ik_w z)\hat{e}_+\right], \tag{A1}$$

where the wiggler magnetic field strength is B_w, the wiggler period is $\ell_w = 2\pi/k_w$ and the basis vectors are $\hat{e}_\pm = (\hat{e}_x \pm i\hat{e}_y)/2$. We have assumed in this model that the beam radius is small compared to the wiggler period $(k_w r_b < 1)$ hence the transverse gradients in the wiggler field are neglected. We similarly invoke the para-axial approximation to the radiation fields and neglect transverse coordinate dependencies in the fields, to obtain the following radiation field model for the vector and scalar potentials,

$$\vec{A}(z,t) = \left[a_f(z,t)\exp(-ik_0 z) + a_b(z,t)\exp(ik_0 z)\right]\exp(i\omega_0 t)\hat{e}_- + \text{c.c.}, \tag{A2}$$

$$\phi(z,t) = \tilde{\phi}(z,t)\exp[-i(k_0 + k_w)z + i\omega_0 t] + \text{ c.c.}, \tag{A3}$$

where $a_f(z,t)$ and $a_b(z,t)$ denote the forward and backward components of the wave field respectively, and $\omega_0 = ck_0$ is the frequency. These field coefficients are assumed to be slowly varying functions of space and time compared to the radiation wavelength and temporal period. The slow spatial dependence of the field coefficients is expressed by, $| Q^{-1}\partial Q/\partial z | << k_0$, with $Q = a_f(z,t), a_b(z,t), \tilde{\phi}(z,t)$ and the slow temporal dependence of the the field coefficients is expressed by, $| Q^{-1}\partial Q/\partial t | << \omega_0$.

The space-time evolution of the fields is governed by Maxwell's equations, which can be cast in the form, $(\partial^2/\partial z^2 - c^{-2}\partial^2/\partial t^2)\vec{A}(z,t) = 4\pi c^{-1}\vec{J}_\perp(z,t)$ and $\partial^2/\partial z^2\phi(z,t) = -4\pi\rho(z,t)$. The driving current and charge densities are obtained from the appropriate moments of the Vlasov distribution function. The Vlasov distribution function is evolved

according to the equation, $\{\partial/\partial t + (p_z/m\gamma)\partial/\partial z - e[\vec{E} + (\vec{p}\times\vec{B})/m\gamma c]\cdot\partial/\partial\vec{p}\}g(z,\vec{p},t) = 0$. By making use of the fact that the canonical transverse momentum is an invariant of the motion and assuming that the beam is cold in the transverse direction (e.g., $g(z, P_x, P_y, p_z, t) = \tilde{g}(z, p_z, t)\delta(P_x)\delta(P_y)$) the evolution of the reduced distribution function is governed by,

$$\left\{\frac{\partial}{\partial t} + \frac{p_z}{m\gamma_T}\frac{\partial}{\partial z} + \left[e\frac{\partial\phi}{\partial z} - \frac{e^2}{2m\gamma_T c^2}\frac{\partial}{\partial z}(\vec{A}\cdot\vec{A})\right]\frac{\partial}{\partial p_z}\right\}\tilde{g}(z, p_z, t) = 0, \tag{A4}$$

where $mc^2\gamma_T = \{m^2c^4 + c^2p_z^2 + e^2(\vec{A}\cdot\vec{A})\}^{1/2}$. Since $\partial(\vec{A}_w\cdot\vec{A}_w)/\partial z = 0$, the equilibrium distribution function satisfies the equation, $\{\partial/\partial t + (p_z/m\gamma_0)\partial/\partial z\}\tilde{g}^{(0)}(z, p_z, t) = 0$, where $mc^2\gamma_0 = \{m^2c^4 + c^2p_z^2 + e^2B_w^2/k_w^2\}^{1/2}$. For long electron beam pulses we shall consider spatially and temporally homogeneous equilibria given by $\tilde{g}^{(0)}(z, p_z, t) = \tilde{g}^{(0)}(p_z)$. To first order in the perturbed fields, the evolution of the linearized distribution function is given by,

$$\left\{\frac{\partial}{\partial t} + \frac{p_z}{m\gamma_0}\frac{\partial}{\partial z}\right\}\tilde{g}^{(1)}(z, p_z, t) = \left[-e\frac{\partial\phi}{\partial z} + \frac{e^2}{m\gamma_0 c^2}\frac{\partial}{\partial z}(\vec{A}\cdot\vec{A}_w)\right]\frac{\partial\tilde{g}^{(0)}}{\partial p_z}. \tag{A5}$$

The solution to the linearized Vlasov equation is formally given by,

$$\begin{aligned}\tilde{g}^{(1)}(z, p_z, t) = \int_0^z dz'\frac{m\gamma_0}{p_z}\Big[&-e\frac{\partial}{\partial z'}\phi(z', t + \frac{(z'-z)}{v_z})\\ &+ \frac{e^2}{m\gamma_0 c^2}\frac{\partial}{\partial z'}(\vec{A}_w(z')\cdot\vec{A}(z', t + \frac{(z'-z)}{v_z}))\Big]\frac{\partial\tilde{g}^{(0)}}{\partial p_z}.\end{aligned} \tag{A6}$$

Linearizing the wave equations for $\vec{A}$ and ϕ one obtains,

$$\begin{aligned}\left(\frac{\partial^2}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A}(z, t) = \frac{-4\pi}{c}\Big\{&\frac{-e^2}{c}\vec{A}_w\int dp_z\frac{\tilde{g}^{(1)}(z, p_z, t)}{m\gamma_0}\\ &- \frac{e^2}{c}\vec{A}\int dp_z\frac{\tilde{g}^{(0)}(p_z)}{m\gamma_0} + \frac{e^2}{c}\vec{A}_w\left(\frac{e^2\vec{A}_w\cdot\vec{A}}{m^2c^4}\right)\int dp_z\frac{\tilde{g}^{(0)}(p_z)}{m\gamma_0^3}\Big\}.\end{aligned} \tag{A7}$$

$$\frac{\partial^2}{\partial z^2}\phi(z, t) = 4\pi n_0 e\int dp_z\tilde{g}^{(1)}(z, p_z, t). \tag{A8}$$

By making use of the slowly varying coefficient approximations, the components of Maxwell's equations can be expressed as:

$$\left\{\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}-\frac{i\omega_p^2}{2\omega_0 c\overline{\gamma}}\left(\alpha^{(1)}-\alpha^{(3)}\frac{\beta_w^2}{2}\right)\right\}a_f(z,t)=$$
$$+\frac{i\omega_p^2}{2\omega_0 c\overline{\gamma}}\frac{B_w}{k_w}\exp[-i(Kz-\omega_0 t)]\int dp_z\frac{\overline{\gamma}}{\gamma_0}\tilde{g}^{(1)}(z,p_z,t), \tag{A9}$$

$$\left\{\frac{\partial}{\partial z}-\frac{1}{c}\frac{\partial}{\partial t}+\frac{i\omega_p^2}{2\omega_0 c\overline{\gamma}}\left(\alpha^{(1)}-\alpha^{(3)}\frac{\beta_w}{2}\right)\right\}a_b(z,t)=0. \tag{A10}$$

$$\left\{\frac{\partial}{\partial z}-\frac{i}{2}K\right\}=i\frac{2\pi n_0 e}{K}\exp[i(Kz-\omega_0 t)]\int dp_z\tilde{g}^{(1)}(z,p_z,t), \tag{A11}$$

where $mc^2\overline{\gamma}=\{m^2c^4+c^2p_{z0}^2+e^2B_w^2/k_w^2\}^{1/2}$, $\alpha^{(n)}=\int dp_z(\overline{\gamma}/\gamma_0)^n\tilde{g}^{(0)}(p_z)$, $\omega_p^2=4\pi n_0e^2/m$, $\beta_w=eB_w/m\overline{\gamma}c^2k_w$ and $K=k_0+k_w$. Also note that in the frequency regime for resonant interaction of the radiation field and the beam particles, $(\omega_0\approx 2\beta_z\gamma_z^2k_wc)$ the driving current for the backward wave is negligible. We shall solve this system of equations in the time asymptotic limit for which the forward wave oscillates at a single complex frequency. $a_f(z,t)=\tilde{a}_f(z)\exp(i\Delta\omega t)$. where $\Delta\omega$ is to be determined self-consistently from the boundary conditions at the mirror surfaces.

The ponderomotive potential in terms of the time asymptotic field coefficients is given by,

$$\vec{A}_w\cdot\vec{A}=\frac{B_w}{2k_w}\tilde{a}_f(z)\exp\{-i[Kz-(\omega_0+\Delta\omega)t]\}+\text{nonresonant terms}. \tag{A12}$$

Retaining only the phase resonant terms, the formal solution to the linearized Vlasov equation can be written as follows,

$$\tilde{g}^{(1)}(z,p_z,t)=\int_0^z dz'\frac{em\gamma_0}{p_z}\exp\left\{-i[K-\frac{(\omega_0+\Delta\omega)}{v_z}]z'\right\}\frac{\partial\tilde{g}^{(0)}(p_z)}{\partial p_z}$$
$$(\partial/\partial z'-iK)\left[\frac{\beta_w\overline{\gamma}}{2\gamma_0}\tilde{a}_f(z')-\tilde{\phi}(z')\right]. \tag{A13}$$

Inserting this result into the Vlasov-Maxwell system of equations one obtains,

$$\left\{\frac{\partial}{\partial z}+i\left[\frac{\Delta\omega}{c}-\frac{\omega_p^2}{2\omega_0 c\overline{\gamma}}(1-\beta_w^2/2)\right]\right\}\tilde{a}_f(z)=$$
$$-i\frac{\omega_p^2}{2\omega_0 c\overline{\gamma}}\frac{\beta_w(1-\beta_{z0})}{\beta_{z0}^3}\frac{(\omega_0+\Delta\omega)}{c}\int_0^z dz'(z'-z)\exp\{-i\Delta K(z'-z)\}$$
$$(\partial/\partial z'-iK)\left[\frac{\beta_w}{2}\tilde{a}_f(z')-\tilde{\phi}(z')\right]+\frac{i\omega_p^2}{2\omega_0 c\overline{\gamma}}\frac{\beta_w}{\beta_{z0}^2}$$
$$\int_0^z dz'\exp\{-i\Delta K(z'-z)\}(\partial/\partial z'-iK)\left[(1+\beta_{z0}^2)\beta_w\tilde{a}_f(z')/2-\tilde{\phi}(z')\right], \qquad (A14)$$

$$\left(\frac{\partial}{\partial z}-\frac{i}{2}K\right)\tilde{\phi}(z)=-\frac{\omega_p^2}{2Kc^2\overline{\gamma}}\frac{(1-\beta_{z0}^2)}{\beta_{z0}^3}\frac{(\omega_0+\Delta\omega)}{c}$$
$$\int_0^z dz'(z'-z)\exp\{-i\Delta K(z'-z)\}(\partial/\partial z'-iK)\left[\beta_w\tilde{a}_f(z')/2-\tilde{\phi}(z')\right]$$
$$+\frac{i\omega_p^2}{2Kc^2\overline{\gamma}}\frac{1}{\beta_{z0}^2}\int_0^z dz'\exp\{-i\Delta K(z'-z)\}(\partial/\partial z'-iK)\left[\beta_w\tilde{a}_f(z')/2-(1-\beta_{z0}^2)\tilde{\phi}(z')\right].$$
$$(A15)$$

where we have defined $\Delta K = K-(\omega_0+\Delta\omega)/v_{z0}$. The previous set of equations yields a dispersion relation which we shall refer to as the complete dispersion relation. A simplified dispersion relation is obtained by noting that in the momentum integration . the results are most sensitive to changes in the exponent, $\Delta K(z'-z)$. Retaining only the terms in the integration by parts which are proportional to $\partial\Delta K/\partial p_z$, one obtains the following simplified system of equations,

$$\left\{\frac{\partial}{\partial z}+i\left[\frac{\Delta\omega}{c}-\frac{\omega_p^2}{2\omega_0 c\overline{\gamma}}(1-\beta_w^2/2)\right]\right\}\tilde{a}_f(z)$$
$$=\frac{-\omega_p^2}{2\omega_0 c\overline{\gamma}}\frac{\beta_w(1-\beta_{z0}^2)}{\beta_{z0}^3}\frac{(\omega_0+\Delta\omega)}{c}\int_0^z dz'(z'-z)\exp\{-i\Delta K(z'-z)\}$$
$$\left(\frac{\partial}{\partial z'}-iK\right)\left[\frac{\beta_w}{2}\tilde{a}_f(z')-\tilde{\phi}(z')\right]. \qquad (A16)$$

$$\left\{\frac{\partial}{\partial z}-\frac{i}{2}K\right\}\tilde{\phi}(z)=-\frac{\omega_p^2}{2Kc^2\overline{\gamma}}\frac{(1-\beta_{z0}^2)}{\beta_{z0}^3}\frac{(\omega_0+\Delta\omega)}{c}$$

$$\int_0^z dz'(z'-z)\exp\{-i\Delta K(z'-z)\}(\frac{\partial}{\partial z'}-iK)[\frac{\beta_w}{2}\tilde{a}_f(z')-\tilde{o}(z')]. \qquad (A17)$$

In both the cases of the complete and simplified set of equations, the equations are of the convolution type and can be solved by Laplace transform methods. The text of the paper consists of a detailed analysis of the simplified set of equations in the Compton regime.

END

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